# Baryon chiral perturbation theory up to next-to-leading order

J.W. Bos[1], D.W. Chang[2], S.C. Lee[3], Y.C. Lin[1], and H.H. Shih[1]
[1] *Department of Physics and Astronomy, National Central University, Chungli, Taiwan*
[2] *Department of Physics, National Tsing Hua University, Hsinchu, Taiwan*
[3] *Institute of Physics, Academia Sinica, Taipei, Taiwan*

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We examine the general lagrangian for baryon chiral perturbation theory with SU(3) flavor symmetry, up to the next-to-leading order. We consider both the strong and the weak interaction. The inverse of the baryon mass is treated as an additional small expansion parameter, and heavy fermion effective field theory techniques are employed to provide a consistent expansion scheme. A detailed account is given on the restrictions imposed on the lagrangian by the various symmetries. Corrections due to the finite baryon mass are also discussed.

## I. INTRODUCTION

Chiral perturbation theory [1] provides a promising way to get insight in low-energy processes involving baryons. In chiral perturbation theory one starts with the most general lagrangian in terms of baryon and meson degrees of freedom, employing the chiral symmetry of the underlying QCD lagrangian in the massless quark limit. Based on this lagrangian, a perturbation scheme is then developed in which one expands in the momenta of the external particles and simultaneously in the mass of the Goldstone bosons.

However, compared to when it is applied to the meson sector [2], chiral perturbation theory applied to the baryon sector is more complicated. In the baryon sector, a loop expansion typically will generate powers of $\dot{m}/\Lambda_{\chi\mathrm{SB}}$, where $\dot{m}$ is the nucleon mass in the chiral limit and $\Lambda_{\chi\mathrm{SB}}$ is the symmetry breaking scale of chiral perturbation theory. Since $\dot{m}/\Lambda_{\chi\mathrm{SB}} \approx 1$, the expansion scheme seems to break down. To avoid this complication, it was suggested [3] to use heavy quark effective field theory techniques—developed originally to treat heavy quark systems—to reformulate baryon chiral perturbation theory with a modified expansion scheme. This is done by redefining the effective baryon field. A loop expansion will now give rise to powers of $k/\Lambda_{\chi\mathrm{SB}}$, where $k$ is a small "residual" nucleon four-momentum, making a systematic expansion feasible. The formulation consists of an simultaneous expansion in $k$, the mass of the strange quark $m_{\mathrm{s}}$ (for simplicity we neglect the up- and down quark masses), and $1/\dot{m}$. We will refer to this formulation as heavy baryon chiral perturbation theory (HBCPT).

In this paper we study the HBCPT lagrangian, with SU(3) flavor symmetry, for both the strong and weak interaction sector. We put emphasis on an essential feature of phenomenological lagrangians, namely that one must include *all* terms consistent with the assumed symmetry properties [4].

In the strong interaction sector, the leading order lagrangian is of order $\mathcal{O}(k)$, i.e., independent of $m_{\mathrm{s}}$, and $\dot{m}$. The next order, or *next-to-leading order*, corrections consist of all terms of order $\mathcal{O}(k^2)$, $\mathcal{O}(m_{\mathrm{s}})$, and $\mathcal{O}(1/\dot{m})$. Note that this classification is only by convention: The relative counting between $k$ and $m_{\mathrm{s}}$ is *a priori* not clear and can only be determined by experiments. In the weak interaction sector, the leading order lagrangian is of order $\mathcal{O}(1)$, and the next-to-leading order lagrangian consists by definition of all terms of order $\mathcal{O}(k)$, $\mathcal{O}(m_{\mathrm{s}})$, and $\mathcal{O}(1/\dot{m})$.

Loop diagrams generated with the leading order HBCPT lagrangian contribute only beyond the next-to-leading order [5], i.e., the most important corrections to a leading-order amplitude come from the next-to-leading order lagrangian at tree-level. This clearly makes it necessary for a consistent calculation to take into account the full structure of the next-to-leading order lagrangian.

This paper is organized as follows. In Sec. II we will discuss the HBCPT lagrangian in the strong interaction sector. First, we will rewrite the leading order lagrangian using the heavy fermion approach. To obtain the next-to-leading order lagrangian we will examine the conditions imposed on a general term by the various symmetry requirements. The possible linear relations between single and double trace terms in the lagrangian is considered closely. We will then give the $1/\dot{m}$-lagrangian, using an approach based on the equation of motion. In Sec. III we look at the weak interaction, and finally, Sec. IV contains a brief summary and our conclusions.

## II. STRONG INTERACTION LAGRANGIAN

### A. Leading order

The leading order strong interaction lagrangian of baryon chiral perturbation theory is given by [6,7]

$$\begin{aligned}\mathcal{L} = \mathrm{Tr}\, \bar{B} i\gamma^\mu [D_\mu, B] - \dot{m}\bar{B}B + D\bar{B}i\gamma^5\gamma^\mu\{\Delta_\mu, B\} \\ + F\bar{B}i\gamma^5\gamma^\mu[\Delta_\mu, B]\,,\end{aligned} \tag{1}$$

where $B$ is the baryon field

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix} , \quad (2)$$

and $\dot{m}$ is the nucleon mass in the chiral limit. The mesons are contained in the anti-hermitian field $\Delta^\mu$, given by

$$\Delta^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger) , \quad (3)$$

where $\xi$ is defined by $\xi^2 = \Sigma$. Here $\Sigma$ is the SU(3) matrix

$$\Sigma = \exp(2i\pi/f_\pi) , \quad (4)$$

with

$$\pi = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \overline{K^0} & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \quad (5)$$

and $f_\pi$ is the pion decay constant, $f_\pi = 94$ MeV. The meson fields also appear in the covariant derivative $D_\mu$, given by

$$D^\mu = \partial^\mu + \Gamma^\mu , \quad (6)$$

with $\Gamma^\mu$ the anti-hermitian field

$$\Gamma^\mu = \frac{1}{2}(\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger) . \quad (7)$$

The operator $D^\mu$ can only appear in the lagrangian in combination with a commutator bracket, i.e. as

$$[D^\mu, B] = \partial^\mu B + [\Gamma^\mu, B] . \quad (8)$$

The chiral symmetry of the lagrangian Eq. (1) is clear, because in the above representation *all* the fields transform under $\mathrm{SU(3)_L} \times \mathrm{SU(3)_R}$ as

$$X \to U X U^\dagger \quad (9)$$

where, given

$$\xi^2 \to \xi'^2 = V_\mathrm{L} \xi^2 V_\mathrm{R}^\dagger \quad \textit{with} \quad V_\mathrm{L}, V_\mathrm{R} \in \mathrm{SU(3)} , \quad (10)$$

$U$ is implicitly defined by

$$\xi' = V_\mathrm{L} \xi U^\dagger = U \xi V_\mathrm{R}^\dagger . \quad (11)$$

We do not consider external fields in our formulation.

Since [6]

$$i\gamma^\mu [D_\mu, B] - \dot{m} B = \mathcal{O}(p) , \quad (12)$$

where $p$ is the nucleon four-momentum, the strong interaction lagrangian Eq. (1) counts as order $\mathcal{O}(p)$ in the momentum expansion. However, there is drastic difference in the behavior of chiral loops iterated with the lagrangian Eq. (1) and the familiar chiral perturbation theory lagrangian in the meson sector. Unlike in the meson sector, in the baryon sector *any* higher-order loop starts to contribute at order $\mathcal{O}(p^2)$, i.e. there is no correspondence between the loop expansion and the momentum expansion [6]. The reason for this is that the nucleon mass in the chiral limit, $\dot{m}$, is comparable with the chiral symmetry breaking scale $\Lambda_{\chi SB}$. To deal with this problem, baryon chiral perturbation theory was reformulated [3]. It starts by redefining the baryon field according to

$$B_v = \mathrm{e}^{i\dot{m} v \cdot x} B , \quad (13)$$

where $v^\mu$ is the baryon four-velocity satisfying $v^2 = 1$. Using

$$i D^\mu B = \mathrm{e}^{-i\dot{m} v\cdot x} (\dot{m} v^\mu + i D^\mu) B_v \quad (14)$$

the lagrangian reads, in terms of the new velocity-dependent baryon field, $B_v$,

$$\begin{aligned} &\mathrm{Tr}\, \bar{B}_v i\gamma^\mu [D_\mu, B_v] - \dot{m} \bar{B}_v (1 - \not{v}) B_v \\ &\quad + D \bar{B}_v i\gamma^5 \gamma^\mu \{\Delta_\mu, B_v\} + F \bar{B}_v i\gamma^5\gamma^\mu [\Delta_\mu, B_v] . \end{aligned} \quad (15)$$

Next, one defines the projected fields

$$B_v^{(+)} = P_v^+ B_v \; ; \;\; B_v^{(-)} = P_v^- B_v , \quad (16)$$

where $P_v^+$ and $P_v^-$ are the projection operators

$$P_v^\pm = \frac{1 \pm \not{v}}{2} . \quad (17)$$

We will show later that the minus component field, $B_v^{(-)}$, is suppressed by $1/\dot{m}$ as compared to the plus component field, $B_v^{(+)}$. Therefore, in leading order in the $1/\dot{m}$ expansion we can drop the minus component field. Using the operators

$$P_v^+ \gamma^\mu P_v^+ = P_v^+ v^\mu \quad (18)$$

and

$$S_v^\mu \equiv P_v^+ \gamma_5 \gamma^\mu P_v^+ , \quad (19)$$

we find that the lowest order HBCPT lagrangian for the strong interaction reads

$$\begin{aligned} \mathcal{L}_v = &\mathrm{Tr}\, \bar{B}_v^{(+)} i v^\mu [D_\mu, B_v^{(+)}] - 2iD \bar{B}_v^{(+)} S_v^\mu \{\Delta_\mu , B_v^{(+)}\} \\ &- 2iF \bar{B}_v^{(+)} S_v^\mu [\Delta_\mu , B_v^{(+)}] . \end{aligned} \quad (20)$$

It can be seen from Eq. (20) that these new baryon fields obey the modified free Dirac equation

$$i v \cdot \partial B_v^{(+)} = 0 , \quad (21)$$

which no longer has the mass term. In momentum space, with $p^\mu$ the four-momentum of the baryon, and using Eq. (13), we see that derivatives of $B_v^{(+)}$ produce powers of

$$k^\mu = p^\mu - \dot{m} v^\mu , \quad (22)$$

which is (for processes at low energies) a small four-momentum. Therefore, the residual baryon momentum, $k$, is the effective expansion parameter in this formulation of baryon chiral perturbation theory, and the lagrangian Eq. (20) counts as $\mathcal{O}(k)$.

### B. Next-to-leading order

#### 1. General lagrangian

A general term in the strong interaction lagrangian of HBCPT—considering only terms relevant to processes involving one baryon—is of the form

$$\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2 \;, \tag{23a}$$

$$\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2 \times \mathrm{Tr}\,A_3 \;, \tag{23b}$$

or

$$\mathrm{Tr}\,\bar{H}A_1 \times \Gamma_v \times \mathrm{Tr}\,A_2 H \times \mathrm{Tr}\,A_3 \;. \tag{23c}$$

Here, and in the following, $H$ is defined as the ($v$-dependent) plus component of the baryon field, i.e.

$$H = B_v^{(+)} \;, \tag{24}$$

$\Gamma_v$ can be any ($v$-dependent) operator in Dirac space, and $A_1$, $A_2$, and $A_3$ can each be any combination of the bosonic fields.

For the fields contained in Eq. (23) we can use $D^\mu$ and $\Delta^\mu$, but also the hermitian scalar field $\sigma$ and the anti-hermitian pseudoscalar field $\rho$, defined by

$$\sigma = \frac{1}{2}(\xi\chi^\dagger\xi + \xi^\dagger\chi\xi^\dagger);\quad \rho = \frac{1}{2}(\xi\chi^\dagger\xi - \xi^\dagger\chi\xi^\dagger) \;, \tag{25}$$

where $\chi$ is the chiral- and $\mathrm{SU(3)_F}$ symmetry breaking mass matrix

$$\chi = B\,\mathrm{diag}(0,0,m_s) \;. \tag{26}$$

(For simplicity we take $m_\mathrm{u} = m_\mathrm{d} = 0$.) Some combinations of $\Delta^\mu$ and $D^\mu$ are related, e.g. one has [7]

$$[D^\mu, D^\nu] = -[\Delta^\mu, \Delta^\nu] \;. \tag{27}$$

Two of the possible operators in Dirac space have been already given by Eqs. (18) and (19). Since

$$P_v^+\gamma_5 P_v^+ = 0 \tag{28}$$

and

$$P_v^+\sigma^{\mu\nu}P_v^+ = -2i[S_v^\mu, S_v^\nu] \;, \tag{29}$$

we then find that $\Gamma_v$ in Eq. (23) can be one of the operators

$$\mathbb{1};\quad P_v^+ v^\mu;\quad S_v^\mu;\quad [S_v^\mu, S_v^\nu] \tag{30}$$

combined with a general tensor constructed from $g^{\mu\nu}$, $\epsilon^{\mu\nu\rho\lambda}$, and $v^\mu$. The latter is generated by the covariant derivative $iD^\mu$ as in Eq. (14). Note that none of the terms in Eq. (30) behaves like a pseudoscalar and $v\cdot S_v = 0$ with $v^2 = 1$.

Unlike in the SU(2) chiral perturbation theory, where the quark mass is usually counted as $\mathcal{O}(k^2)$, we have in SU(3) chiral perturbation theory simultaneous expansions in the independent variables $k$ and $m_\mathrm{s}$. The leading order lagrangian Eq. (20) is of order $\mathcal{O}(k)$. In *next-to-leading order* we then have all the terms of order $\mathcal{O}(k^2)$ and $\mathcal{O}(m_\mathrm{s})$. Note that it is a matter of definition to count the $\mathcal{O}(m_\mathrm{s})$ terms as next-to-leading order. In the heavy baryon formulation, we expect in general to have in addition an expansion in $\mathcal{O}(1/\dot{m})$. This will be discussed later.

To obtain the chiral power of a given term in the lagrangian we use that the fields $\Delta^\mu$ and $D^\mu$ count as order $\mathcal{O}(k)$, while the fields $\sigma$ and $\rho$ count as order $\mathcal{O}(m_\mathrm{s})$. All the matrices in Dirac space count as order $\mathcal{O}(1)$.

To arrive at the next-to-leading order strong lagrangian we follow similar lines as in Ref. [7]. We will demand that a given term in the lagrangian is Lorentz invariant, space-reversal invariant, charge conjugation invariant, and hermitian.

#### 2. Terms in the lagrangian with one trace

There are two possible terms of the form Eq. (23a), with aside from the two baryon fields *one* additional field, A. In order to study the charge conjugation invariance and hermiticity it is convenient to write these two terms in the form

$$\mathrm{Tr}\,\bar{H}\Gamma_v\{A,H\}\;;\quad \mathrm{Tr}\,\bar{H}\Gamma_v[A,H] \;. \tag{31}$$

In the following we will use the short notation

$$\big(A_1, A_2\big) = \{A_1, A_2\}\ \mathit{or}\ [A_1, A_2] \tag{32}$$

for the (anti)commutator brackets, i.e., Eq. (31) is equivalent to

$$\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big) \;. \tag{33}$$

Under the charge conjugation operation the baryon field behaves as

$$H \to C\bar{H}^\mathrm{T} \;, \tag{34}$$

where $C$ is the charge conjugation operator, in our representation given by $C = i\gamma^0\gamma^2$. Since $H = P_v^+ \mathrm{e}^{i\dot{m}v\cdot x}B$, we obtain for the charge conjugated baryon field

$$H^\mathrm{c} = P_v^- \mathrm{e}^{i\dot{m}(-v)\cdot x}B^\mathrm{c} \;, \tag{35}$$

where $B^\mathrm{c} = C\bar{B}^\mathrm{T}$. Therefore, the charge conjugation operation on a bilinear term in the baryon fields reads

$$\big[\bar{H}\Gamma_v H\big]^\mathrm{c} = \bar{H}^\mathrm{c}\Gamma_{-v}H^\mathrm{c} \;. \tag{36}$$

For the terms of the form Eq. (33) we then have under charge conjugation

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\right]^{\mathrm{c}} = \mathrm{Tr}\,\bar{H}^{\mathrm{c}}\Gamma_{-v}\big(A^{\mathrm{c}},H^{\mathrm{c}}\big) = (-1)^{c_{\Gamma_v}+c_A}\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\ , \quad (37)$$

where the constants $c_{\Gamma_v}$ and $c_A$ are implicitly defined by

$$A^{\mathrm{c}} = (-1)^{c_A}A^{\mathrm{T}};\ \ C^{-1}\Gamma_{-v}C = (-1)^{c_{\Gamma_v}}\Gamma_v^{\mathrm{T}}\ . \quad (38)$$

In Table I we display for all fields the constants $c_A$, and in Table II we display for all operators in Dirac space the constants $c_{\Gamma_v}$.

Under complex conjugation we have for terms of the form Eq. (33)

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\right]^{*} = \mathrm{Tr}\,\bar{H}\gamma^0\Gamma_v^{\dagger}\gamma^0\big(A^{\dagger},H\big) = (-1)^{h_{\Gamma_v}+h_A}\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\ . \quad (39)$$

The constants $h_A$ and $h_{\Gamma_v}$ in Eq. (39) are defined by

$$A^{\dagger} = (-1)^{h_A}A;\ \ \gamma_0\Gamma_v^{\dagger}\gamma_0 = (-1)^{h_{\Gamma_v}}\Gamma_v\ . \quad (40)$$

Again, these constants can be found in Tables I and II.

The properties of the fields and bilinear products of the Dirac matrices under Lorentz transformations are well known (see e.g. Ref. [7]), and are summarized in Tables I and II.

From charge conjugation invariance and Eq. (37) it follows that we have terms of the form of Eq. (33) *only if* $c_{\Gamma_v}+c_A$ *is even.* Hermiticity can easily be established by using Eq. (39) and by multiplying a term by the complex number $i$, if $h_{\Gamma_v}+h_A$ is odd. Lorentz- and space-reversal invariance of terms is secured by. contracting all the free Lorentz indices with appropriate tensors.

One readily finds that, except for the terms already encountered in the leading order lagrangian Eq. (20), the only allowed terms with one field are the two SU(3) breaking terms

$$i\mathrm{Tr}\,\bar{H}\big(\sigma,H\big)\ . \quad (41)$$

The terms in Eq. (41) are of order $\mathcal{O}(m_{\mathrm{s}})$ and therefore belong, in our classification scheme, to the next-to-leading order lagrangian.

We will now consider all possible terms of the form Eq. (23a) containing *two* bosonic fields, aside from the baryon fields. Closer inspection shows that the six possible combinations can be written as

$$\mathrm{Tr}\,\bar{H}\Gamma_v\big({}^1A_1,\big({}^2A_2,H^2\big)^1\big);\quad \mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\ . \quad (42)$$

Below it will become clear that this is a convenient way of writing the terms down. We have used the superscripts 1 and 2 in the first term of Eq. (42) in order to make the distinction between the (anti)commutator brackets associated with the fields $A_1$ and $A_2$. From now we will assume it is clear that these brackets can be different, and we will therefore drop the superscript. We will also assume that in a equation where a given field appears in two terms, as in Eq. (43) below, the bracket associated with that field is the same in both terms. Under charge conjugation the terms in Eq. (42) behave as

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A_1,\big(A_2,H\big)\big)\right]^{\mathrm{c}} = (-1)^{c_{\Gamma_v}+c_{A_1}+c_{A_2}} \times \mathrm{Tr}\,\bar{H}\Gamma_v\big(A_2,\big(A_1,H\big)\big) \quad (43a)$$

and

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\right]^{\mathrm{c}} = (-1)(-1)^{c_{\Gamma_v}+c_{A_1}+c_{A_2}} \times \mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\ , \quad (43b)$$

while under complex conjugation we have for such terms

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A_1,\big(A_2,H\big)\big)\right]^{*} = (-1)^{h_{\Gamma_v}+h_{A_1}+h_{A_2}} \times \mathrm{Tr}\,\bar{H}\Gamma_v\big(A_2,\big(A_1,H\big)\big) \quad (44a)$$

and

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\big)\right]^{*} = (-1)(-1)^{h_{\Gamma_v}+h_{A_1}+h_{A_2}} \times \mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\ . \quad (44b)$$

It can be seen from Eq. (43), that the allowed terms containing two fields, $A_1$ and $A_2$, are given by

$$\mathrm{Tr}\,\bar{H}\Gamma_v\big(A_1,\big(A_2,H\big)\big) + \mathrm{Tr}\,\bar{H}\Gamma_v\big(A_2,\big(A_1,H\big)\big) \quad (45)$$

if $c_{\Gamma_v}+c_{A_1}+c_{A_2}$ is *even*, and given by

$$\mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big) \quad (46)$$

if $c_{\Gamma_v}+c_{A_1}+c_{A_2}$ is *odd*. Hermiticity can easily be established by using Eq. (44). Combining the building blocks of Tables I and II we then find that the allowed terms with two fields, up to next-to-leading order, are

$$\mathrm{Tr}\,\bar{H}[v\cdot D,[v\cdot D,H]] \quad (47a)$$
$$\mathrm{Tr}\,\bar{H}[D^{\mu},[D_{\mu},H]] \quad (47b)$$
$$\mathrm{Tr}\,\bar{H}[S_v^{\mu},S_v^{\nu}]\big([D_{\mu},D_{\nu}],H\big) \quad (47c)$$
$$\mathrm{Tr}\,\bar{H}S_v^{\mu}[D_{\mu},\big(v\cdot\Delta,H\big)] + \mathrm{Tr}\,\bar{H}S_v^{\mu}\big(v\cdot\Delta,[D_{\mu},H]\big) \quad (47d)$$
$$\mathrm{Tr}\,\bar{H}S_v^{\nu}[v\cdot D,\big(\Delta_{\nu},H\big)] + \mathrm{Tr}\,\bar{H}S_v^{\nu}\big(\Delta_{\nu},[v\cdot D,H]\big) \quad (47e)$$
$$\mathrm{Tr}\,\bar{H}\big(\Delta^{\mu},\big(\Delta_{\mu},H\big)\big) \quad (47f)$$
$$\mathrm{Tr}\,\bar{H}\big(v\cdot\Delta,\big(v\cdot\Delta,H\big)\big)\ . \quad (47g)$$

Since $\{A,[A,B]\} = [A,\{A,B\}]$, only three of the four terms in Eqs. (47f) and (47g) are independent. All the terms in Eq. (47) are of order $\mathcal{O}(k^2)$ in the expansion. It can be easily seen that the terms in Eqs. (47d) and (47e) couple an *odd* number of mesons to the baryon, while the other terms in Eq. (47) couple an *even* number.

### *3. Terms in the lagrangian with more than one trace*

Now we turn to the general terms with more than one trace in flavor space, given by Eqs. (23b) and (23c).

For terms of the form Eq. (23b) we have under charge conjugation,

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2 \times \mathrm{Tr}\,A_3\right]^{\mathrm{c}} = \left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2\right]^{\mathrm{c}} \times \mathrm{Tr}\,A_3^{\mathrm{c}} \ , \qquad (48)$$

while under complex conjugation we have

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2 \times \mathrm{Tr}\,A_3\right]^{*} = \left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H A_2\right]^{*} \times \mathrm{Tr}\,A_3^{\dagger} \ . \qquad (49)$$

The right-hand side of Eq. (48) can be obtained using Eqs. (37) and (43), and the right-hand side of Eq. (49) can be obtained using Eqs. (39) and (44). Since

$$\mathrm{Tr}\,\Delta^\mu = 0 \ ; \quad \mathrm{Tr}\,[D^\mu, X] = 0 \ \textit{for any traceless field } X \ , \qquad (50)$$

it is easily established that, up to next-to-leading order, all possible terms of the form Eq. (23b) are

$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\sigma \qquad (51a)$$
$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\Delta\cdot\Delta \qquad (51b)$$
$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,(v\cdot\Delta)^2 \ . \qquad (51c)$$

For terms of the form Eq. (23c) we have under charge conjugation and complex conjugation

$$\left[\mathrm{Tr}\,\bar{H}A_1 \times \Gamma_v \times \mathrm{Tr}\,A_2 H \times \mathrm{Tr}\,A_3\right]^{\mathrm{c}} = (-1)^{c_{\Gamma_v}+c_{A_1}+c_{A_2}+c_{A_3}}\,\mathrm{Tr}\,\bar{H}A_2 \times \Gamma_v \times \mathrm{Tr}\,A_1 H \times \mathrm{Tr}\,A_3 \qquad (52)$$

and

$$\left[\mathrm{Tr}\,\bar{H}A_1 \times \Gamma_v \times \mathrm{Tr}\,A_2 H \times \mathrm{Tr}\,A_3\right]^{*} = (-1)^{h_1+h_2+h_3+h_{\Gamma_v}}\,\mathrm{Tr}\,\bar{H}A_2 \times \Gamma_v \times \mathrm{Tr}\,A_1 H \times \mathrm{Tr}\,A_3 \ , \qquad (53)$$

respectively. It then easily follows that all these are given by

$$\mathrm{Tr}\,\bar{H}\Delta^\mu \times \mathrm{Tr}\,\Delta_\mu H \qquad (54a)$$
$$\mathrm{Tr}\,\bar{H}\Delta_\mu \times [S_v^\mu, S_v^\nu] \times \mathrm{Tr}\,\Delta_\nu H \qquad (54b)$$
$$\mathrm{Tr}\,\bar{H}v\cdot\Delta \times \mathrm{Tr}\,v\cdot\Delta H \ . \qquad (54c)$$

Not all the double trace terms in the lagrangian are independent due to the matrix relation by Cayley: For four traceless $3\times 3$ matrices, $A$, $B$, $C$, and $D$, one has

$$\mathrm{Tr}\big(DABC + DCAB + DACB + DBAC + DBCA + DCBA\big) = \mathrm{Tr}\,DC \times \mathrm{Tr}\,AB + \mathrm{Tr}\,DB \times \mathrm{Tr}\,AC + \mathrm{Tr}\,DA \times \mathrm{Tr}\,BC \ . \qquad (55)$$

Cayley's identity, Eq. (55), can be rewritten in a form more suitable for our application as

$$\mathrm{Tr}\,DC \times \mathrm{Tr}\,AB = \frac{3}{4}\mathrm{Tr}\left(D\{A,\{B,C\}\} + D\{B,\{A,C\}\}\right) + \frac{1}{4}\mathrm{Tr}\left(D[A,[B,C]] + D[B,[A,C]]\right) - \mathrm{Tr}\,DB \times \mathrm{Tr}\,AC - \mathrm{Tr}\,DA \times \mathrm{Tr}\,BC \ . \qquad (56)$$

Using Eq. (56) and the fact that $\bar{H}$, $H$, and $\Delta^\mu$ are traceless $3\times 3$ matrices in flavor space, one finds

$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\Delta\cdot\Delta = \frac{3}{2}\mathrm{Tr}\,\bar{H}\{\Delta^\mu,\{\Delta_\mu,H\}\} + \frac{1}{2}\mathrm{Tr}\,\bar{H}[\Delta^\mu,[\Delta_\mu,H]] - 2\mathrm{Tr}\,\bar{H}\Delta^\mu \times \mathrm{Tr}\,\Delta_\mu H \ . \qquad (57)$$

and

$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,(v\cdot\Delta)^2 = \frac{3}{2}\mathrm{Tr}\,\bar{H}\{v\cdot\Delta,\{v\cdot\Delta,H\}\} + \frac{1}{2}\mathrm{Tr}\,\bar{H}[v\cdot\Delta,[v\cdot\Delta,H]] - 2\mathrm{Tr}\,\bar{H}v\cdot\Delta \times \mathrm{Tr}\,v\cdot\Delta H \ . \qquad (58)$$

Therefore the double trace terms in Eqs. (51b) and (51c) are in fact a linear combination of terms in Eqs. (47) and (54). Note that the above relations are also very useful for large $N_{\mathrm{c}}$ analysis.

Counting all the independent terms in Eqs. (41), (47), (51), and (54), we find that the the next-to-leading order lagrangian in the strong interaction sector consists of 20 terms. Of these 20 terms, 17 terms are of order $\mathcal{O}(k^2)$, and 3 terms are of order $\mathcal{O}(m_{\mathrm{s}})$.

### C. $1/\mathring{m}$ corrections of the strong lagrangian

Based on the equation of motion approach of Ref. [5], we now examine the first order correction to the HBCPT lagrangian due to the finite nucleon mass. To arrive at the HBCPT lagrangian Eq. (20) the small component of the baryon field, $B_v^{(-)}$, has been dropped. To examine this approximation further, we go back to the lagrangian in Eq. (15) obtained after redefining the baryon field. We first rewrite it as

$$\mathrm{Tr}\,\bar{B}_v G(B_v) - \mathring{m}\bar{B}_v(1-\not{v})B_v \ , \qquad (59)$$

where

$$G(B_v) \equiv i\gamma^\mu[D_\mu, B_v] + Di\gamma^5\gamma^\mu\{\Delta_\mu, B_v\} + Fi\gamma^5\gamma^\mu[\Delta_\mu, B_v] \ . \qquad (60)$$

It follows from Eq. (59) that the velocity dependent baryon field, $B_v$, satisfies the equation of motion

$$\mathring{m}(1-\not{v})B_v = G(B_v) \ . \qquad (61)$$

Multiplying Eq. (61) on the left by the projection operator $P_v^-$ gives

$$B_v^{(-)} = \frac{1}{2\mathring{m}}P_v^- G(B_v) \ . \qquad (62)$$

Since $B_v = B_v^{(-)} + B_v^{(+)}$ it follows by iteration that

$$B_v^{(-)} = \frac{1}{2\mathring{m}}P_v^- G\left(B_v^{(+)}\right) + \mathcal{O}(1/\mathring{m}^2) \ . \qquad (63)$$

Also, the equation of motion Eq. (61) implies

$$P_v^+ G(B_v) = P_v^+ G\left(B_v^{(+)} + B_v^{(-)}\right) = 0 \,. \tag{64}$$

Substituting Eq. (63) for $B_v^{(-)}$ in Eq. (64) leads to

$$P_v^+ G\left(B_v^{(+)} + \frac{1}{2\dot{m}} P_v^- G\left(B_v^{(+)}\right)\right) = 0 \,, \tag{65}$$

which can be interpreted as the equation of motion for the $B_v^{(+)}$ field up to first order in $1/\dot{m}$. We conclude from this that the lagrangian of baryon chiral perturbation theory up to order $\mathcal{O}(1/\dot{m})$ reads

$$\mathrm{Tr}\, \bar{B}_v^{(+)} G\left(B_v^{(+)} + \frac{1}{2\dot{m}} P_v^- G\left(B_v^{(+)}\right)\right) \,, \tag{66}$$

which can be rewritten as

$$\mathrm{Tr}\, \bar{B}_v^{(+)} G\left(B_v^{(+)}\right) + \frac{1}{2\dot{m}} \mathrm{Tr}\, \bar{B}_v^{(+)} G\left(P_v^- G\left(B_v^{(+)}\right)\right) \,. \tag{67}$$

The first term in the right-hand side of Eq. (67) corresponds to the leading order lagrangian $\mathcal{L}_v$ given by Eq. (20), and the second term in the right-hand side of Eq. (67) is the $1/\dot{m}$ correction lagrangian

$$\mathcal{L}^{1/\dot{m}} = \frac{1}{2\dot{m}} \mathrm{Tr}\, \bar{B}_v^{(+)} G\left(P_v^- G\left(B_v^{(+)}\right)\right) \,. \tag{68}$$

Using Eqs. (60) and (68), and again writing $H \equiv B_v^{(+)}$, we find that the $1/\dot{m}$ lagrangian is given by

$$\begin{aligned}\mathcal{L}^{1/\dot{m}} = \frac{1}{2\dot{m}} \mathrm{Tr}\, \bar{H}\Big(&[v\cdot D, [v\cdot D, H]] \\ &- [D^\mu, [D_\mu, H]] + 2[S_v^\mu, S_v^\nu][D_\mu, [D_\nu, H]] \\ &- 2DS_v^\mu\left([D_\mu, \{v\cdot\Delta, H\}] + \{v\cdot\Delta, [D_\mu, H]\}\right) \\ &- 2FS_v^\mu\left([D_\mu, [v\cdot\Delta, H]] + [v\cdot\Delta, [D_\mu, H]]\right) \\ &+ DF\left(\{v\cdot\Delta, [v\cdot\Delta, H]\} + [v\cdot\Delta, \{v\cdot D, H\}]\right) \\ &+ D^2\{v\cdot\Delta, \{v\cdot\Delta, H\}\} + F^2[v\cdot\Delta, [v\cdot\Delta, H]]\Big) \,. \end{aligned} \tag{69}$$

Two different approaches—based on reparametrization-invariance [8] and using path integrals, respectively—lead to the same result for the $1/\dot{m}$ lagrangian.

It can be seen that the terms in $\mathcal{L}^{1/\dot{m}}$, Eq. (69), form a subset of the terms in the general lagrangian of order $\mathcal{O}(k^2)$, given in the previous section. Since the coefficients of the higher order terms in the lagrangian will all have to be determined by the experimental data one may question the usefulness of calculating the $1/\dot{m}$ correction to the leading order lagrangian. In HBCPT there are two independent small masses, $k$ and $m_s$, and two independent large masses, $\Lambda_{\chi SB}$ and $\dot{m}$. In terms of extracting information from the data and then drawing predictions from the result, one only has to control the expansion in the small mass parameters. It is not essential which large masses take up the role of balancing the dimensions. Therefore, unless one can show that the combination of terms in Eq. (69) plays a special role in the next-to-leading order corrections, it is for a phenomenological fitting not necessary to take into account $1/\dot{m}$ corrections from the lower order lagrangian.

## III. WEAK INTERACTION

The leading order $|\Delta \mathrm{S}| = 1$ weak lagrangian in chiral perturbation theory is given by [9]

$$\mathcal{L}_{\mathrm{W}} = h_D \mathrm{Tr}\, \bar{B}\{\lambda, B\} + h_F \mathrm{Tr}\, \bar{B}[\lambda, B] \,, \tag{70}$$

where

$$\lambda \equiv \xi^\dagger \lambda_6 \xi \,. \tag{71}$$

The field $\lambda$ is constructed as in Eq. (71) in order to make the chiral symmetry of the lagrangian Eq. (70) manifest and to satisfy the $(8_{\mathrm{L}}, 1_{\mathrm{R}})$ transformation property of the weak interaction.

### A. Leading order weak lagrangian and $1/\dot{m}$ corrections

We now proceed in the same way as in the previous section to obtain the weak lagrangian, including $1/\dot{m}$ corrections, in HBCPT. Redefining the baryon field as in Eq. (13), the weak lagrangian Eq. (70) becomes

$$\mathrm{Tr}\, \bar{B}_v G_{\mathrm{W}}(B_v) \,, \tag{72}$$

where

$$G_{\mathrm{W}}(B_v) \equiv h_D\{\lambda, B_v\} + h_F[\lambda, B_v] \,. \tag{73}$$

Following the steps leading to Eq. (67) we find that the weak lagrangian, including the $1/\dot{m}$ correction, reads

$$\mathrm{Tr}\, \bar{H} G_{\mathrm{W}}(H) + \frac{1}{2\dot{m}} \mathrm{Tr}\, \bar{H} G_{\mathrm{W}}\left(P_v^- G_{\mathrm{W}}(H)\right) \,. \tag{74}$$

However, in this case it can be shown that the second term in the right-hand side of Eq. (74) vanishes, i.e., there are *no* $1/\dot{m}$ corrections in the weak sector, and the weak HBCPT lagrangian simply reads

$$\mathcal{L}_{v,\mathrm{W}} = h_D \mathrm{Tr}\, \bar{H}\{\lambda, H\} + h_F \mathrm{Tr}\, \bar{H}[\lambda, H] \,. \tag{75}$$

The $1/\dot{m}$ corrections usually are associated with only the spin-flip part of an interaction. The vanishing of the $1/\dot{m}$ terms can then be understood since in the weak sector the interaction is spin independent—the structure in Dirac space is just the unity matrix.

### B. The next-to-leading order weak lagrangian

In the weak interaction both the invariance under charge conjugation, $C$, and space-reversal, $P$, is violated. However, to a great accuracy the theory is invariant under their combined action, $CP$. In order to obtain the terms allowed in the general lagrangian, we investigate the behavior of a given term under $CP$. Of course, we also will demand Lorentz invariance and hermiticity. General terms in the weak lagrangian again have the form as given by Eq. (23).

#### 1. Terms in the weak lagrangian with one trace

First, we consider the case of a term with only *one* field. All possible terms with one field are of the form in Eq. (33). Under charge conjugation one has for such terms (see Eq. (37))

$$\begin{aligned}\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\right]^{\mathrm{c}} &= \mathrm{Tr}\,\bar{H}^{\mathrm{c}}\Gamma_v\big(A^{\mathrm{c}},H^{\mathrm{c}}\big)\\ &= (-1)^{c_{\Gamma_v}}\,\mathrm{Tr}\,\bar{H}\Gamma_v\Big(\left(A^{\mathrm{c}}\right)^{\mathrm{T}},H\Big)\;, \end{aligned}\tag{76}$$

where $c_{\Gamma_v}$ is defined by Eq. (38). Next, the parity operation gives,

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A,H\big)\right]^{\mathrm{cp}} = \mathrm{Tr}\,\bar{H}\hat{\Gamma}_v\big(\hat{A},H\big)\;,\tag{77}$$

where we defined

$$\hat{\Gamma}_v = (-1)^{c_{\Gamma_v}} P^{-1}\Gamma_{\tilde{v}}P\;;\quad \hat{A} = \Big(\left(A^{\mathrm{c}}\right)^{\mathrm{T}}\Big)^{\mathrm{P}}\;,\tag{78}$$

with $P$ the parity operator, in our representation given by $P=\gamma^0$, and $\tilde{v}^\mu \equiv v_\mu$. Similarly, for terms containing *two* fields, given by Eq. (45), we have under $CP$

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big(A_1,\big(A_2,H\big)\big)\right]^{\mathrm{cp}} = \mathrm{Tr}\,\bar{H}\hat{\Gamma}_v\big(\hat{A}_2,\big(\hat{A}_1,H\big)\big)\tag{79a}$$

and

$$\left[\mathrm{Tr}\,\bar{H}\Gamma_v\big([A_1,A_2],H\big)\right]^{\mathrm{cp}} = -\mathrm{Tr}\,\bar{H}\hat{\Gamma}_v\big([\hat{A}_1,\hat{A}_2],H\big)\;,\tag{79b}$$

respectively. To establish hermiticity for such terms we can make use of Eqs. (39) and (44).

As building blocks for terms in the general weak lagrangian we have to our disposal the fields $\lambda$, $\Delta^\mu$, $D^\mu$, $\sigma$, $\rho$, and the field $\lambda'$ defined by

$$\lambda' = \xi^\dagger\lambda_7\xi\;,\tag{80}$$

because $\lambda_7$ also induces s$\rightarrow$d transitions. Note that any term must contain the field $\lambda$ or $\lambda'$, to insure the correct $(8_{\mathrm{L}},1_{\mathrm{R}})$ transformation property. For the operators in Dirac space we may take the same as in the strong interaction case. In Tables III and IV we have displayed the necessary properties of the fields and operators needed to apply Eqs. (77) and (79). In order to consider hermiticity we use

$$\lambda^\dagger = \lambda;\;\;(\lambda')^\dagger = \lambda'\;.\tag{81}$$

It easily follows that the $CP$-even terms with one bosonic field are

$$\mathrm{Tr}\,\bar{H}\big(\lambda,H\big)\;,\tag{82}$$

i.e., indeed as in the leading order weak lagrangian $\mathcal{L}_{v,\mathrm{W}}$, Eq. (75). Note that terms of the form Eq. (82) with $\lambda'$ instead of $\lambda$ are $CP$-odd.

Finally, the $CP$-even terms with two bosonic fields are

$$i\mathrm{Tr}\,\bar{H}\big(\lambda,[v\cdot D,H]\big) + i\mathrm{Tr}\,\bar{H}[v\cdot D,\big(\lambda,H\big)]\tag{83a}$$
$$i\mathrm{Tr}\,\bar{H}S_v^\mu\big(\lambda,[D_\mu,H]\big) + i\mathrm{Tr}\,\bar{H}S_v^\mu[D_\mu,\big(\lambda,H\big)]\tag{83b}$$
$$i\mathrm{Tr}\,\bar{H}\big(\lambda,\big(v\cdot\Delta,H\big)\big) + i\mathrm{Tr}\,\bar{H}\big(v\cdot\Delta,\big(\lambda,H\big)\big)\tag{83c}$$
$$i\mathrm{Tr}\,\bar{H}S_v^\mu\big(\lambda,\big(\Delta_\mu,H\big)\big) + i\mathrm{Tr}\,\bar{H}S_v^\mu\big(\Delta_\mu,\big(\lambda,H\big)\big)\tag{83d}$$
$$\mathrm{Tr}\,\bar{H}\big([v\cdot D,\lambda'],H\big)\tag{83e}$$
$$\mathrm{Tr}\,\bar{H}S_v^\mu\big([D_\mu,\lambda'],H\big)\tag{83f}$$
$$\mathrm{Tr}\,\bar{H}\big([v\cdot\Delta,\lambda'],H\big)\tag{83g}$$
$$\mathrm{Tr}\,\bar{H}S_v^\mu\big([\Delta_\mu,\lambda'],H\big)\tag{83h}$$
$$\mathrm{Tr}\,\bar{H}\big(\lambda,\big(\sigma,H\big)\big) + \mathrm{Tr}\,\bar{H}\big(\sigma,\big(\lambda,H\big)\big)\tag{83i}$$
$$i\mathrm{Tr}\,\bar{H}\big(\lambda',\big(\rho,H\big)\big) + i\mathrm{Tr}\,\bar{H}\big(\rho,\big(\lambda',H\big)\big)\tag{83j}$$
$$i\mathrm{Tr}\,\bar{H}\big([\lambda',\sigma],H\big)\tag{83k}$$
$$\mathrm{Tr}\,\bar{H}\big([\lambda,\rho],H\big)\;.\tag{83l}$$

The terms in Eq. (83a) to (83h) are of order $\mathcal{O}(k)$ while the terms in Eq. (83i) to (83l) are of order $\mathcal{O}(m_{\mathrm{s}})$. Note that the parity of a given term in Eq. (83) is indefinite.

#### 2. Terms in the weak lagrangian with more than one trace

We now turn to the general terms in the weak interaction lagrangian with more than one trace in flavor space, given by Eqs. (23b) and (23c). For terms of the form Eq. (23b) we have under the combined charge conjugation and parity operation

$$\begin{aligned}&\left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H\times\mathrm{Tr}\,A_2\right]^{\mathrm{cp}}\\ &\quad= \left[\mathrm{Tr}\,\bar{H}\Gamma_v A_1 H\right]^{\mathrm{cp}}\times\left[\mathrm{Tr}\,A_2\right]^{\mathrm{cp}}\;,\end{aligned}\tag{84}$$

which can be obtained by using Eqs. (77) and (79), and

$$\left[\mathrm{Tr}\,A_3\right]^{\mathrm{cp}} = \mathrm{Tr}\,\hat{A}_3\;.\tag{85}$$

For the behavior of these terms under complex conjugation we can make use of Eq. (49). Taking into account Eq. (50) it is easily established that possible $CP$-even terms of the form Eq. (23b) in the weak sector are

$$i\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda v\cdot\Delta \tag{86a}$$
$$i\mathrm{Tr}\,\bar{H}S_v^\mu H \times \mathrm{Tr}\,\lambda\Delta_\mu \tag{86b}$$
$$\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda\sigma \tag{86c}$$
$$i\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda'\rho \tag{86d}$$
$$\mathrm{Tr}\,\bar{H}\big(\lambda, H\big) \times \mathrm{Tr}\,\sigma \tag{86e}$$
$$i\mathrm{Tr}\,\bar{H}\big(\lambda', H\big) \times \mathrm{Tr}\,\rho\ . \tag{86f}$$

For term of the form Eq. (23c) we have under a $CP$ transformation

$$\begin{aligned}\big[\mathrm{Tr}\,\bar{H}A_1 \times \Gamma_v \times \mathrm{Tr}\,A_2 H \times \mathrm{Tr}\,A_3\big]^{\mathrm{cp}} = \\ \mathrm{Tr}\,\bar{H}\hat{A}_2 \times \hat{\Gamma}_v \times \mathrm{Tr}\,\hat{A}_1 H \times \mathrm{Tr}\,\hat{A}_3\ ,\end{aligned} \tag{87}$$

and for complex conjugation we can use Eq. (53). It then easily follows that all the $CP$-even terms of the form Eq. (23c) are given by

$$i\mathrm{Tr}\,\bar{H}\lambda \times \mathrm{Tr}\,v\cdot\Delta H + i\mathrm{Tr}\,\bar{H}v\cdot\Delta \times \mathrm{Tr}\,\lambda H \tag{88a}$$
$$i\mathrm{Tr}\,\bar{H}\lambda \times S_v^\mu \times \mathrm{Tr}\,\Delta_\mu H + i\mathrm{Tr}\,\bar{H}\Delta_\mu \times S_v^\mu \times \mathrm{Tr}\,\lambda H \tag{88b}$$
$$\mathrm{Tr}\,\bar{H}\lambda' \times \mathrm{Tr}\,v\cdot\Delta H - \mathrm{Tr}\,\bar{H}v\cdot\Delta \times \mathrm{Tr}\,\lambda' H \tag{88c}$$
$$\begin{aligned}&\mathrm{Tr}\,\bar{H}\lambda' \times S_v^\mu \times \mathrm{Tr}\,\Delta_\mu H \\ &\quad - \mathrm{Tr}\,\bar{H}\Delta_\mu \times S_v^\mu \times \mathrm{Tr}\,\lambda' H\end{aligned} \tag{88d}$$
$$\mathrm{Tr}\,\bar{H}\lambda \times \mathrm{Tr}\,\sigma H + \mathrm{Tr}\,\bar{H}\sigma \times \mathrm{Tr}\,\lambda H \tag{88e}$$
$$i\mathrm{Tr}\,\bar{H}\lambda' \times \mathrm{Tr}\,\rho H + i\mathrm{Tr}\,\bar{H}\rho \times \mathrm{Tr}\,\lambda' H \tag{88f}$$
$$i\mathrm{Tr}\,\bar{H}\lambda' \times \mathrm{Tr}\,\sigma H - i\mathrm{Tr}\,\bar{H}\sigma \times \mathrm{Tr}\,\lambda' H \tag{88g}$$
$$\mathrm{Tr}\,\bar{H}\lambda \times \mathrm{Tr}\,\rho H - \mathrm{Tr}\,\bar{H}\rho \times \mathrm{Tr}\,\lambda H\ . \tag{88h}$$

As in the strong interaction case, some of the double trace terms in the weak interaction lagrangian are related to the single trace terms. Applying Cayley's identity, Eq. (56), one obtains

$$\begin{aligned}&i\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda v\cdot\Delta = \\ &\quad \frac{3i}{4}\big(\mathrm{Tr}\,\bar{H}\{\lambda,\{v\cdot\Delta, H\}\} + \mathrm{Tr}\,\bar{H}\{v\cdot\Delta,\{\lambda, H\}\}\big) \\ &\quad + \frac{i}{4}\big(\mathrm{Tr}\,\bar{H}[\lambda,[v\cdot\Delta, H]] + \mathrm{Tr}\,\bar{H}[v\cdot\Delta,[\lambda, H]]\big) \\ &\quad - i\mathrm{Tr}\,\bar{H}\lambda \times \mathrm{Tr}\,v\cdot\Delta H - i\mathrm{Tr}\,\bar{H}v\cdot\Delta \times \mathrm{Tr}\,\lambda H\ ,\end{aligned} \tag{89}$$

i.e., the double-trace term in Eq. (86a) can be written as a combination of the double-trace term in Eq. (88a) and two of the single-trace terms in Eq. (83c). In the same way we have for the term in Eq. (86b)

$$\begin{aligned}&i\mathrm{Tr}\,\bar{H}S_v^\mu H \times \mathrm{Tr}\,\lambda\Delta_\mu = \\ &\quad \frac{3i}{4}\big(\mathrm{Tr}\,\bar{H}S_v^\mu\{\lambda,\{\Delta_\mu, H\}\} + \mathrm{Tr}\,\bar{H}S_v^\mu\{\Delta_\mu,\{\lambda, H\}\}\big) \\ &\quad + \frac{i}{4}\big(\mathrm{Tr}\,\bar{H}S_v^\mu[\lambda,[\Delta_\mu, H]] + \mathrm{Tr}\,\bar{H}S_v^\mu[\Delta_\mu,[\lambda, H]]\big) \\ &\quad - i\mathrm{Tr}\,\bar{H}\lambda \times S_v^\mu \times \mathrm{Tr}\,\Delta_\mu H - i\mathrm{Tr}\,\bar{H}\Delta_\mu \times S_v^\mu \times \mathrm{Tr}\,\lambda H\ .\end{aligned} \tag{90}$$

Since $\sigma$ and $\rho$ are non-traceless matrices in flavor space, we cannot apply Cayley's identity for the double-trace terms in Eqs. (86c) and (86d). However, it can be easily shown that for three $3\times3$ traceless matrices $A$, $C$, and $D$, and one arbitrary $3\times3$ matrix $B$, one has

$$\begin{aligned}&\mathrm{Tr}\,DC \times \mathrm{Tr}\,AB = \\ &\quad \frac{3}{4}\mathrm{Tr}\,\big(D\{A,\{B,C\}\} + D\{B,\{A,C\}\}\big) \\ &\quad + \frac{1}{4}\mathrm{Tr}\,\big(D[A,[B,C]] + D[B,[A,C]]\big) \\ &\quad - \mathrm{Tr}\,DB \times \mathrm{Tr}\,AC - \mathrm{Tr}\,DA \times \mathrm{Tr}\,BC \\ &\quad - \mathrm{Tr}\,D\{A,C\} \times \mathrm{Tr}\,B\ .\end{aligned} \tag{91}$$

Using Eq. (91) we then find that the terms in Eqs. (86c) and (86d) satisfy

$$\begin{aligned}&\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda\sigma = \\ &\quad \frac{3}{4}\big(\mathrm{Tr}\,\bar{H}\{\lambda,\{\sigma, H\}\} + \mathrm{Tr}\,\bar{H}\{\sigma,\{\lambda, H\}\}\big) \\ &\quad + \frac{1}{4}\big(\mathrm{Tr}\,\bar{H}[\lambda,[\sigma, H]] + \mathrm{Tr}\,\bar{H}[\sigma,[\lambda, H]]\big) \\ &\quad - \mathrm{Tr}\,\bar{H}\lambda \times \mathrm{Tr}\,\sigma H - \mathrm{Tr}\,\bar{H}\sigma \times \mathrm{Tr}\,\lambda H \\ &\quad - \mathrm{Tr}\,\bar{H}\{\lambda, H\} \times \mathrm{Tr}\,\sigma\ ,\end{aligned} \tag{92}$$

and

$$\begin{aligned}&i\mathrm{Tr}\,\bar{H}H \times \mathrm{Tr}\,\lambda'\rho = \\ &\quad \frac{3i}{4}\big(\mathrm{Tr}\,\bar{H}\{\lambda',\{\rho, H\}\} + \mathrm{Tr}\,\bar{H}\{\rho,\{\lambda', H\}\}\big) \\ &\quad + \frac{i}{4}\big(\mathrm{Tr}\,\bar{H}[\lambda',[\rho, H]] + \mathrm{Tr}\,\bar{H}[\rho,[\lambda', H]]\big) \\ &\quad - i\mathrm{Tr}\,\bar{H}\lambda' \times \mathrm{Tr}\,\rho H - i\mathrm{Tr}\,\bar{H}\rho \times \mathrm{Tr}\,\lambda' H \\ &\quad - i\mathrm{Tr}\,\bar{H}\{\lambda', H\} \times \mathrm{Tr}\,\rho\ ,\end{aligned} \tag{93}$$

respectively, i.e., they can both be written as a linear combination of terms in Eqs. (83), (86), and (88).

In total we find in the next-to-leading order $CP$-even weak interaction lagrangian 44 independent terms, of which 24 are of order $\mathcal{O}(k)$ and 20 of order $\mathcal{O}(m_{\mathrm{s}})$. The $CP$-odd lagrangian can be easily obtained from the $CP$-even lagrangian by exchanging in every term $\lambda$ by $\lambda'$.

## IV. SUMMARY AND CONCLUSIONS

In this paper we have considered the general lagrangian of baryon chiral perturbation theory in the heavy baryon formulation for the strong- and weak interaction sector up to next-to-leading order, i.e., the first order correction to the leading order. Using general symmetry principles we gave the restrictions on a given term in the lagrangian. We investigated the relation between the terms with a single and a double trace in flavor space. In the strong interaction sector we found a total number of 20 terms in the next-to-leading order lagrangian, while that number in the weak interaction sector is 44. In the weak interaction sector we only gave explicitly the $CP$-even part; the extension to the $CP$-odd part is trivial.

We have also examined the $1/\dot{m}$ corrections. In the weak interaction sector these are shown to be absent. This is due to the simple structure of the weak interaction in Dirac space. The significance of this observation is unclear yet. In the strong interaction sector the $1/\dot{m}$ lagrangian is a linear combination of terms in the next-to-leading order lagrangian. For this reason we argued that it is in fact not necessary to take $1/\dot{m}$ corrections into account when doing a phenomenological fitting.

For a consistent study of low-energy processes with baryons, e.g. hyperon decay, it is necessary to include all the terms in the lagrangian in the calculation. Starting with this lagrangian, one can then fix by experimental data as many of the coefficients associated with the terms in the lagrangian as possible. Once the coefficients are fixed, new predictions can be drawn. This work is in progress. Also, we have considered only the octet baryons so far. In general, the decuplet may also play a important role in some of the phenomenological issues. However, inclusion of the decuplet involves some complications which will be resolved later.


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TABLE I. Properties of the fields, needed for building general terms in the baryon chiral lagrangian in the strong interaction sector. The constants $c_A$ and $h_A$, in the second and third column respectively, are defined by Eqs. (38) and (40), respectively. The fourth column gives, in the usual notation, the properties under the Lorentz and parity transformation.

| $A$ | $c_A$ | $h_A$ | L |
|---|---|---|---|
| $\Delta^\mu$ | 0 | 1 | A |
| $D^\mu$ | 1 | 1 | V |
| $\sigma$ | 0 | 0 | S |
| $\rho$ | 0 | 1 | PS |

TABLE II. Same as Table I for the operators in Dirac space. They all count as order $\mathcal{O}(1)$ in the chiral expansion. We only write down the operators with at most two Lorentz indices, since operators with three or more indices will contribute only to higher order terms.

| $\Gamma_v$ | $c_{\Gamma_v}$ | $h_\Gamma$ | L [a] |
|---|---|---|---|
| $\mathbb{1}$ | 0 | 0 | S |
| $P_v^+ v^\mu$ | 1 | 0 | V |
| $S_v^\mu$ | 0 | 0 | A |
| $[S_v^\mu, S_v^\nu]$ | 1 | 1 | T |
| $P_v^+ v^\mu v^\nu$ | 0 | 0 | T |
| $S_v^\mu v^\nu$ | 1 | 0 | PT |

[a] It is here assumed that the matrices are in combination with the bilinear product of the baryon fields

TABLE III. Properties of the fields, needed to consider the combined operation of charge conjugation and parity on a general term in the weak lagrangian. Given a field $A$, the field $\hat{A}$ is defined by Eq. (78).

| $A$ | $\hat{A}$ |
|---|---|
| $\lambda$ | $\lambda$ |
| $\lambda'$ | $-\lambda'$ |
| $\Delta^\mu$ | $-\Delta_\mu$ |
| $D^\mu$ | $-D_\mu$ |
| $\sigma$ | $\sigma$ |
| $\rho$ | $-\rho$ |

TABLE IV. As Table III for the operators in Dirac space. Again, given a operator $\Gamma_v$, the fields $\hat{\Gamma}_v$ is defined by Eq. (78). No operators with more than one Lorentz index are needed, since they will only appear in terms of higher order in the weak lagrangian.

| $\Gamma_v$ | $\hat{\Gamma}_v$ |
|---|---|
| $v^\mu$ | $-v_\mu$ |
| $S_v^\mu$ | $-(S_v)_\mu$ |